

How to hand-calculate MTF in front-side and backside illuminated image sensors

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Our purpose was to create a closed-form analytical $MTF_{nyquist}$ model, being suitable for integration in a spreadsheet-like calculator

Advantages of this analytical model	Weak points of this analytical model
- Offers insight - Spreadsheet like what-if analysis, - Trend predicting capabilities - Closed form expression for LSF and $MTF_{nyquist}$	- Simplified, “homogenous boxes” geometry 2D projection of 3D reality - Front side metal pattern is not modeled - Effect of recombination is not modeled

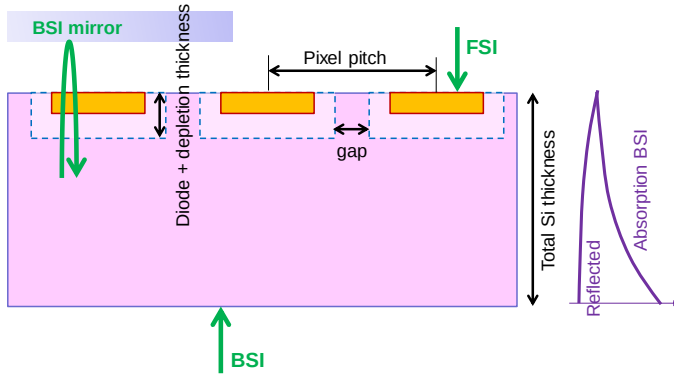


Figure 1 “homogenous boxes” approximation of pixel configuration and design parameters considered

An intermediate “single depth MTF_1 ” is the product of 2 factors, which correspond to the convolution of the key contributors of the Line Spread Function:

$$MTF_1 = MTF_{diodsheet} \cdot MTF_{diffusion}$$

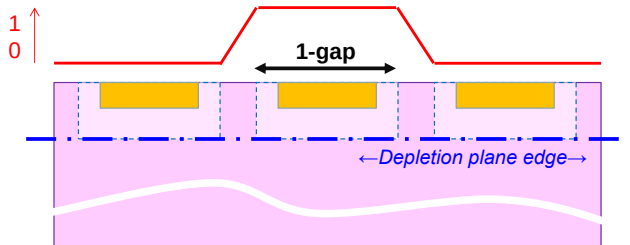


Figure 2 LSF (red) of the boxed collecting photodiode sheet. “1-gap” is the width of the collecting junction including its lateral depletion layers, as a fraction of the pixel pitch.

References

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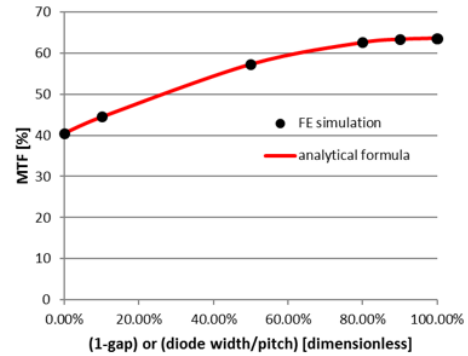


Figure 2 Finite elements simulation versus analytical expression for $MTF_{diodsheet}$.

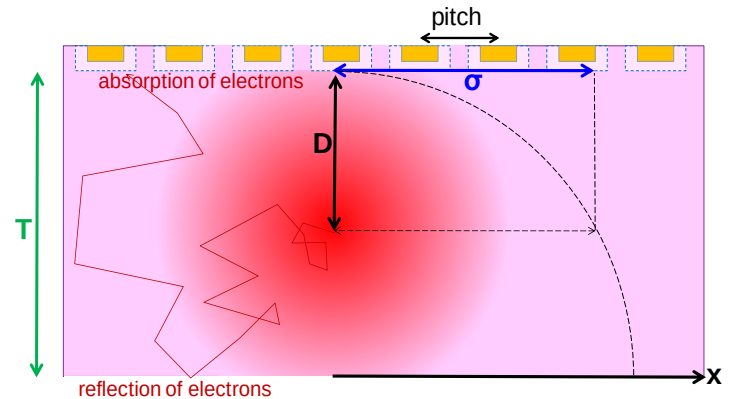


Figure 3 Diffusion of photoelectrons in a sheet of Silicon, between the (absorbing) depletion edge plane and the (reflecting) bottom plane.

The [probability] distribution $P(x)$, hence $LSF_{diffusion}$, for an electron starting at depth D to arrive at the depletion plane edge at position x is found to be ^[2,3,4]

$$P(x) = \frac{\frac{1}{2T} \cdot \sin\left(\frac{\pi D}{T}\right)}{\cosh\left(\frac{\pi x}{T}\right) - \cos\left(\frac{\pi D}{T}\right)} + \frac{\frac{1}{T} \cdot \sin\left(\frac{\pi D}{2T}\right) \cdot \left[\cosh\left(\frac{\pi x}{2T}\right) - \cos\left(\frac{\pi D}{2T}\right)\right]}{\cosh\left(\frac{\pi x}{T}\right) - \cos\left(\frac{\pi D}{T}\right)}$$

The expression is (and should be) geometrically invariant: it is a function of x/T and D/T . Trivial cases are found for $T=D$ and $D=0$. The MTF of this distribution subsequently is derived as

$$MTF_{diffusion} = \text{norm.dist}(\pi \cdot \sigma / \text{pitch}) \quad \text{wherein } \sigma^2 = T^2 - (T-D)^2,$$

The overall MTF is obtained as the integral of all MTF_1 s for all depths using the absorption relation of the wavelength chosen.

