How to hand-calculate MTF in front-side and backside illuminated image sensors

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Our purpose was to create a closed-form analytical $MTF_{nyquist}$ model, being suitable for integration in a spreadsheet-like calculator, enabling thus quick surveys and design parameter trade-offs in the presence of many other image sensor parameters. Solutions that require CPU-intensive numerical calculations or iterative solutions are not suitable for that purpose. The method yields the Line Spread Function as intermediate result.

Prior models for MTF prediction were based on "brute force" solving the diffusion equations in a finite elements mesh detailing the pixel's geometry. This approach has potentially the best match with reality. In fact, we used this approach to "calibrate" our numerical model. More advanced approaches as [1] strongly reduce the calculation burden by expressing the MTF of a point source, and then integrating that over the total geometry.

Advantages of this analytical model	Weak points of this analytical model
• Offers insight • Spreadsheet like what-if analysis, • Trend predicting capabilities • Closed form expression for LSF and $MTF_{nyquist}$	• Simplified, "boxed" geometry, homogenous regions • 2D projection of 3D reality • Front side metal pattern is not modeled • Effect of recombination is not modeled

In order to come to a model that is at the same time trend predicting, and offering insight in the actual diffusion and optical crosstalk mechanisms, we had to simplify the geometrical topology of the pixel further as in Figure 1.

An intermediate "single depth" MTF_1 is the product of 2 factors, which corresponds to the convolution of the key contributors of the Line Spread Function:

$$MTF_1 = MTF_{diodesheet} \cdot MTF_{diffusion}$$

Whereby

a) **MTF_1** is the $MTF_{nyquist}$, assuming that all photoelectrons emerge at one single depth in the Silicon.

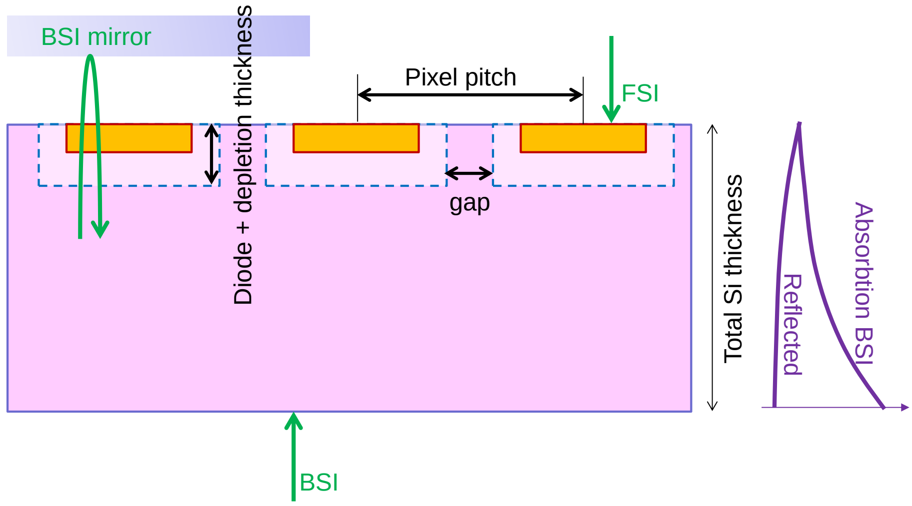

Figure 1 cross section of pixel configuration and design parameters considered.

b) **MTF$_{\text{diodesheet}}$** is the MTF of the sheet of charge collecting photodiodes. We simplify the geometry to a 2-dimensional projection of box-shaped implants (Figure 2). The pixel's line spread function closely approximates a trapezoidal function. One can then prove that following analytical expression closely approximates the MTF:

$$\text{MTF}_{\text{diodesheet}} = \left(\frac{2}{\pi}\right).\,sinc_{\pi}\left(\frac{1-gap}{2}\right) = \left(\frac{2}{\pi}\right)^2.\frac{\sin(\frac{\pi}{2}.(1-gap))}{1-gap}$$

If the photoelectrons are generated inside this layer, we assume that $\text{MTF}_1 = \text{MTF}_{\text{diodesheet}}$

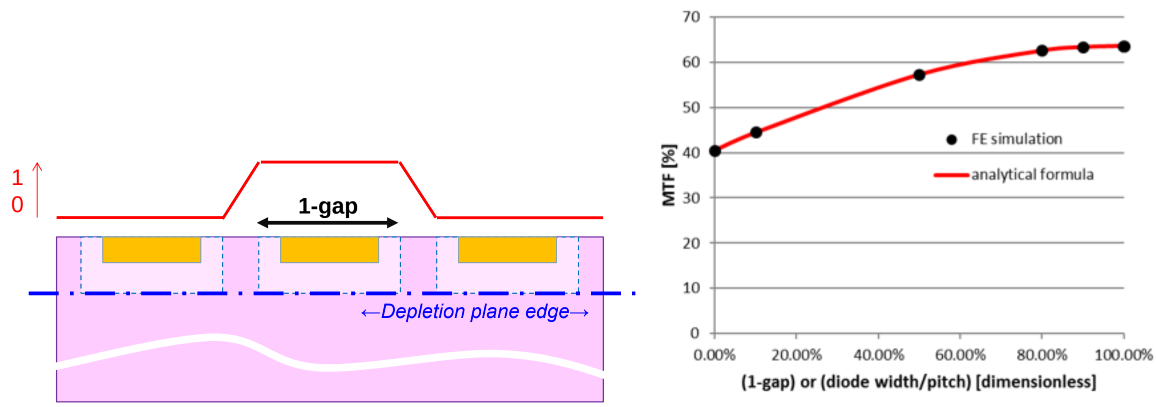

Figure 2 Left: LSF of the simplified collecting photodiode sheet. "1-gap" is the width of the collecting junction including its lateral depletion layers, as a fraction of the pixel pitch. Right: finite elements simulation and analytical expression for MTF$_{\text{diodesheet}}$.

c) **MTF$_{\text{diffusion}}$** is the MTF due to diffusion of photocharges generated below the depletion edge plane. Assuming that the depletion edge plane is a perfect absorber of charges and that the BSI surface (or epi layer to bulk edge) is a perfect reflector of charges, we could derive a mathematically exact closed form expression [2,3,4] for the distribution P(x) of electrons arriving at the depletion edge plane, being thus the Line Spread Function for a point source:

$$P(x) = \frac{\frac{1}{2T}\cdot sin\left(\frac{\pi D}{T}\right)}{cosh\left(\frac{\pi x}{T}\right) - cos\left(\frac{\pi D}{T}\right)} + \frac{\frac{1}{T}\cdot sin\left(\frac{\pi D}{2T}\right)\cdot\left[cosh\left(\frac{\pi x}{2T}\right) - cos\left(\frac{\pi D}{2T}\right)\right]}{cosh\left(\frac{\pi x}{T}\right) - cos\left(\frac{\pi D}{T}\right)}$$

Wherein T is the layer thickness and the electron generation happens at coordinate (0,D), as explained in Figure 3. The MTF of this distribution subsequently is derived as

$MTF_{diffusion}$=norm.dist(π.σ/pitch) wherein $\sigma^2=T^2-(T-D)^2$, which is exact as far as P(x) is exactly a Gaussian distribution and a good approximation in other cases as found in FE simulations.

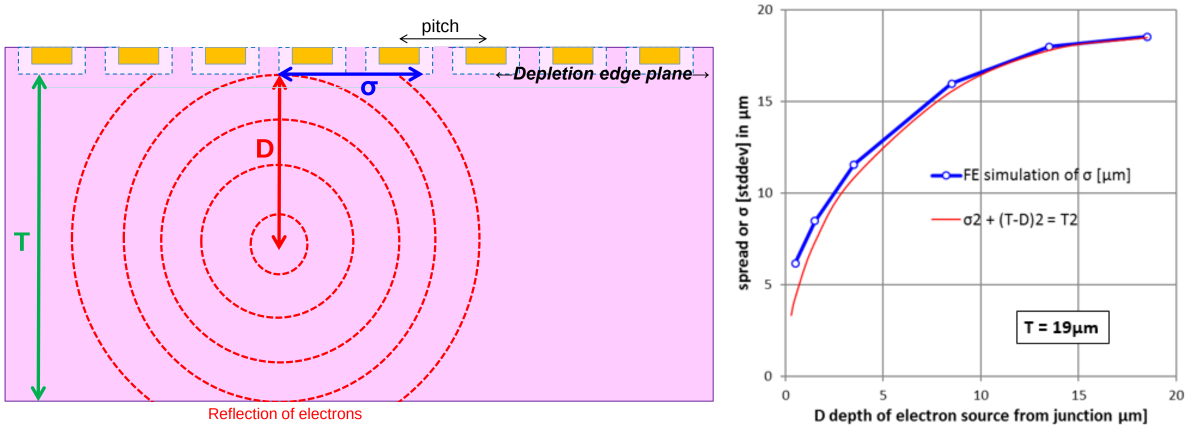

Figure 3 diffusion of photoelectrons in a sheet of Silicon, between the (absorbing) depletion edge plane and the (reflecting) bottom plane. Right: FE simulation and analytical expression of the distribution's σ as function of D (versus a fixed T). The 0.5µm shift in D-direction is due to the granularity of the FE solver.

In order to come to an overall aggregate MTF, we must

- For a given wavelength of light, integrate the MTF_1 over all depths, using that wavelength's absorption length as a weighting factor. We consider in our spreadsheets the cases BSI, FSI and BSI with a reflector.
- For a given illuminant, integrate over the wavelength range, using the (illuminant's spectrum * QE) as weighting factor.

References

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