ISSCC Forum on Image sensors
3 February 2008

High Dynamic Range
the pixel standpoint

Bart Dierickx
Caeleste CVBA
Antwerp, Belgium
www.caeleste.be
bart@caeleste.be
1. Why do we need a wide dynamic range? ✓
   Examples of dynamic range scenes, extreme contrast scenes and what we would like to see in them

2. Dynamic range and S/N definitions ✓
   There is not a single definition for S, N, and even less for DR. We base the definitions on “noise equivalent contrast” (NEC)

3. Non-linear response ✓
   As a way to capture the highest possible dynamic range scenes. Derive which non-linear laws are optimal.
1. Why do we need a wide dynamic range?

Wide dynamic range scenes
Extreme contrast scenes
Why do we need a wide dynamic range?

- To catch highlights
- To allow us to be lazy and not adjust camera speed to the scene
- To discriminate objects in any part (dark/bright) of the scene

- Natural scenes
- Extreme contrast scenes
High lights

Dark object With some contrast

Deep dark background
Highlight partly overexposed

In the shadow of a shadowed scene
Deep shadow
No recovery

Heavy overexposure
No recovery
100 dB scene

(courtesy Cypress Semiconductor)

140 dB scene

100 dB scene

??? dB scene


120 dB scene
… and while grabbing a wide dynamic range scene…

Don’t throw away the “sensitivity”

BTW what does sensitivity mean:
• High conversion ratio of light power to voltage?
• To see a faint signal in a limited exposure time?
• To see a faint signal in an unlimited exposure time?
• The ratio of the signal and the uncertainty thereof?
2. dynamic range definitions
Dynamic Range definition?

DR_{wikipedia}

*Wikipedia:* “Dynamic range is a term used frequently in numerous fields to describe the *ratio* between the smallest and largest possible values of a changeable quantity, such as in *sound* and *light*."

Applies to the scene, not to the sensor

- our “changeable quantity” is “P”, “light” [W, W/m2, photons, lux…]
- “signal”, “S”, is the measurement result [V, ADC bits…]
prior definitions

Units:
- W
- Photons/s
- Lumen
- Candela
- W/m²
- Photons/s.m²
- Lumen/m²
- lux

For white light Between 400-700nm
1 W ≈ 150...250 Lumen

For white light Between 400-700nm
1 lux * µm² ≈ 4000 Photons/s
1 lumen ≈ 4E15 Photons/s
1 W ≈ 8e17 photons/s
From scene to focal plane

Natural scenes

Outdoor scene
Indoor bright
Indoor shadow
Sun, Sky,

Focal plane

optical attenuation
divide by $4F^2$

light power [$\text{W/m}^2$]

- Indoor shadow
- Indoor bright
- Outdoor scene
- Sun, Sky,

Focal plane

$F = \frac{f}{D}$
With a linear response sensor

**Linear response sensor:**
S/N or SNR = Dynamic Range?
- typical: Between 1000: 1 = 60 dB
- extreme high end: 10000:1 = 80 dB

Dynamic range in sunlit scene: > 100 dB

\[ \text{S} \approx 1 \text{V}, \text{N} \approx 1 \text{mV}_{\text{RMS}} \]
\[ \text{S} \approx 2 \text{V}, \text{N} \approx 200 \mu\text{V}_{\text{RMS}} \]

**Dynamic range definition, attempt 1**
“DR is in light power domain what \( \frac{S_{\text{max}}}{N_{\text{min}}} \) is in voltage (signal) domain”
Further definitions

- **Illuminant (illuminance)**
  - $W/m^2$
  - Photons/s.m$^2$
  - Lumen/m$^2$=lux

- **Object reflection (emittance)**
  - $1m^2$

- **Lamp surface (emittance)**
  - $1m^2$

- **Scene**

- **Pixel sees**
  - $P [W]$
  - [Photons/s]
  - [Lumen]

- **Photodiode**
  - $I_{photo} [A]$
  - $Q_{signal} [F, e^-]$

- **Signal S**
  - $V_{signal} [V]$
  - ADCbits

- **Transconductance**
  - $C_{eff}$

- **SR, QE**

- **Charge amplifier**

- **Charge Q noise**

- **Signal noise** $V_{noise}$

- **Pixel**

- **Signal processing** $PR$

- **Noise** $Q_{noise}$

- **Integration time** $t_{int}$
DR = S/N ?

In a linear system, Dynamic Range == \( S_{\text{max}} / N_{\text{min}} \)
Maybe there are S/N definitions suitable for non-linear systems?

- **Signal, S, \( V_{\text{signal}} [V] \)**
  - The maximum signal: \( S_{\text{max}}, V_{\text{max}} - V_{\text{dark}} \)
  - At the working point, referred to Dark: \( V - V_{\text{Dark}} \)
  - Linearized around working point
  - Take into account \( \partial V / \partial \text{Illumination} \) or PR?

- **Noise, N, \( V_{\text{noise}} [V_{\text{RMS}}] \)**
  - At dark: \( N_{\text{min}} \)
  - Where S/N == 1
  - At the working point
What is part of N?

- **Spatial noises**
  - Fixed Pattern Noise
  - Temperature dependent: Dark signal non-uniformity (DSNU)
  - Signal dependent: Photo response non-uniformity (PRNU)

- **Temporal noises**
  - kTC, EMI, thermal and 1/f device noise, …
  - Temperature dependent: Dark current shot noise (DCSN)
  - Signal dependent: Photon Shot Noise (PSN)

**Issue:** some of these can be / are calibrated

→ Maximal noise: all of these
→ Minimal noise: only PSN and DCSN
## Noise breakdown

in 6 orthogonal categories

<table>
<thead>
<tr>
<th>Noise that is invariant for temperature, integration time and illumination level</th>
<th>Temporal noise (variation of the signal of one pixel over time)</th>
<th>Spatial noise (variation signals of pixels within a frame, steady over time)</th>
</tr>
</thead>
</table>
| Noise that depends on temperature and integration time | Temporal noise  
- kTC noise, reset noise  
- other pixel and circuit noise  
- EMI (random EMI, Row noise, Interference…)  
- ADC noise | Fixed pattern noise  
- Random FPN  
- Column/Row FPN  
- Other cosmetic flaws  
- EMI fixed interference patterns |
| Noise that depends on the illumination level or signal level | DCSN (dark current shot noise) | DSNU (Dark signal non-uniformity) |
| | PSN (photon shot noise) | Photo response non-uniformity  
- Random PRNU  
- Column PRNU  
- Other cosmetic flaws  
- Color PRNU |
Dynamic Range definition?

\[ \text{DR}_{\text{wikipedia}} \]

- Wikipedia: “Dynamic range is a term used frequently in numerous fields to describe the ratio between the smallest and largest possible values of a changeable quantity, such as in sound and light.”

Applies to the scene, not to the sensor
The sensor should be able to catch that range
What does that mean?
Generalized dynamic range definition, attempt 2

$\text{DR}_{\text{SNR}_1} =$ “ratio between highest and lowest light intensity for which S/N is greater than or equal to 1”.
Dynamic Range definitions

Further attempts for definition

- The range of light intensity levels that can be captured by the image sensor within a single frame
- The range of illumination levels on a similar object within the same frame, for which the object is recognizable (=decent contrast, after image processing)
- The range of intensities that can be captured, for which the SNR has at least a certain value
- The range of intensities that can be captured for which the Noise Equivalent Contrast (NEC) has at least a certain value
### Ratios between the measured quantity and the uncertainty on that measurement

<table>
<thead>
<tr>
<th>definition</th>
<th>Symbol</th>
<th>Unit</th>
<th>How to obtain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal to Noise Ratio</td>
<td>$S/N_{\text{max}}$  \ $\text{SNR}_{\text{max}}$</td>
<td>1</td>
<td>sensor signal voltage range / sensor signal noise in the dark</td>
</tr>
<tr>
<td>Differential or small-signal signal to noise ratio</td>
<td>$\frac{dS}{dN}$ \ $d\text{SNR}$</td>
<td>1</td>
<td>signal voltage / signal noise at that same signal level</td>
</tr>
<tr>
<td>Noise equivalent contrast ratio</td>
<td>NEC</td>
<td>1</td>
<td>The ability to discriminate between nearby grey levels = $1/(d\text{SNR})^*PR$ (where $PR$=photo response)</td>
</tr>
<tr>
<td>Dynamic range</td>
<td>$DR_{\text{max}}$</td>
<td>1</td>
<td>Saturation intensity divided by noise equivalent intensity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>In a linear system this is the same as $\text{SNR}_{\text{max}}$.</td>
</tr>
<tr>
<td>Generalized dynamic range</td>
<td>$DR_{\text{SNR}1}$</td>
<td>1</td>
<td>the ratio between upper and lower intensities for which $d\text{SNR} \geq [\text{value}]$</td>
</tr>
<tr>
<td>Generalized dynamic range</td>
<td>$DR_{\text{NEC10}}$</td>
<td>1</td>
<td>the ratio between upper and lower intensities for which NEC $\geq [\text{value}]$</td>
</tr>
<tr>
<td>Linear dynamic range</td>
<td>$L\text{DR}_x$</td>
<td>1</td>
<td>$DR_x$ with largest intensity for which $d\text{Volt}/d\text{Intensity}$ is linear</td>
</tr>
<tr>
<td>ADC bits</td>
<td></td>
<td>1</td>
<td>Number of (useful) bits in the sensor’s digital output</td>
</tr>
<tr>
<td>bits</td>
<td></td>
<td>1</td>
<td>Number of bits after image processing</td>
</tr>
</tbody>
</table>

Disclaimer: these suffixes are a clarification for this course. They are not used in practice.
Constant N.E. Contrast - linear

Log Signal S,
Noise N
\([V, V_{\text{RMS}}]\)

NEC = SNR

Hypothetical noise for a constant noise equivalent contrast

Read noise

Photon shot noise

Illumination \(P\)
\([W, W.s, W/m^2, \text{lux, photons...}]\)
In a non-linear system $\text{SNR} \neq \text{NEC}$

Log

Signal $S$, Noise $N$ $[V, V_{\text{RMS}}]$

Log

Illumination $P$ $[W, W.s, W/\text{m2}, \text{lux}, \text{photons}...]$

Signal saturates

Total noise / photo response?

SNR is finite?

Read noise

Total noise
Noise Equivalent Contrast - general

\[ NEC = \frac{P}{NEP} \]

\[ NEP = \frac{N}{Photoresponse} = \frac{N}{\left(\frac{\partial S}{\partial P}\right)} \]

\[ NEC = \frac{P}{NEP} = \frac{P}{N} \cdot \frac{\partial S}{\partial P} \]
Noise Equivalent Contrast – non-linear

\[ NEC = \frac{P}{NEP} \]

\[ NEP = \frac{N}{\text{Photoresponse}} = \frac{N}{\left( \frac{\partial S}{\partial P} \right)} \]

\[ NEC = \frac{P}{NEP} = \frac{P \cdot \frac{\partial S}{\partial P}}{N} \]

(In a linear system NEC == S/N)

\[ V_{signal} = S \]
\[ V_{noise} = N \]
Relation between NEC and Wide DR

- Goal: reach a constant or minimal NEC over the largest possible [dynamic range] \text{wikipedia definition}

- Unproven underlying hypothesis: the largest range is obtained when NEC is just large enough, i.e. constant

\[
NEC = \frac{P}{NEP} = \frac{P \cdot \partial S}{\partial P} = \text{constant}
\]
In search for a constant NEC

- **Exercise of thought:**
  - Obtain the constant NEC by exploiting non-linear response
    - Increase DR by sacrificing NEC where it is sufficient
  - Non-linear response is obtained by
    - A non-linear transconductance, gain or $C_{\text{effective}}$
    - A non-linear integration time $t_{\text{int}}$
  - In the presence of noise of following kinds
    - A fixed amount in charge or light (as kTC…)
    - A fixed amount in voltage (other read noise, circuit noise)
    - Proportional to the $\sqrt{\text{power}}$ (photon shot noise, PSN)
    - Proportional to the $\sqrt{\text{integration time}}$ (dark current shot noise, DCSN)
    - Proportional to the power (photo response non-uniformity, PRNU)
    - Proportional to the integration time (Dark signal non-uniformity, DSNU)
Is ISO a potential measure for dynamic range?

<table>
<thead>
<tr>
<th>method</th>
<th>Formula</th>
<th>Border conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMS Bristol’s</td>
<td>ISO = 0.2 [W.s/m²] / (t_int[s] * P[W/m²])</td>
<td>White light, P is average of scene, 50% of saturation</td>
</tr>
<tr>
<td>Basler’s</td>
<td>100 ISO corresponds to 0.1 Lux*sec</td>
<td>Where the average P is at 18% of a linear grey scale</td>
</tr>
<tr>
<td>Michael Kriss’</td>
<td>ISO Speed = 0.8/H</td>
<td>at a signal-to-noise ratio of 30</td>
</tr>
<tr>
<td>Interpreting ISO12232</td>
<td>ISO == (1.92 lx.s)/Lf.t</td>
<td>at 18%...50% of saturation - the level of saturation itself is a parameter that one may change</td>
</tr>
<tr>
<td></td>
<td>Lf [lx] = focal plane luminance</td>
<td></td>
</tr>
<tr>
<td>Kodak’s &quot;saturation based&quot; ISO</td>
<td>ISO = (15.4 * f²) / (Ls * t)</td>
<td>Ls[lx] = scene luminance 18/106 saturation (18/170 for professional photography)</td>
</tr>
<tr>
<td>Kodak’s &quot;noise based&quot; ISO</td>
<td>ISO = 10 / H</td>
<td>SNR is either 40 (excellent image) or 10 (acceptable image), all noises included</td>
</tr>
<tr>
<td></td>
<td>H = exposure in [lx.s]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Does not account for color information</th>
<th>Does not account for image processing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does not account for effect of resolution on subjective perception</td>
<td>Does not account for dynamic range</td>
</tr>
</tbody>
</table>
3. non-linearity

non-linear response as a way to increase the sensor’s capability to capture a wide dynamic range scene

non-linear response as a way to exploit the fact that the noise level depends on the scene contents
“dynamic range > 80 dB cannot be reached with a linear response sensor” – so what?

\[ V_{\text{signal}} = \frac{t_{\text{int}}}{C_{\text{eff}}} \cdot I_{\text{photo}} \]

\[ V_{\text{signal}} = \frac{Q_{\text{signal}}}{C_{\text{eff}}} \]

\[ V_{\text{noise}} = \frac{Q_{\text{noise}}}{C_{\text{eff}}} \]

**Modulation of the integration time**
- Multiple slope response (piece-wise linear slopes)
- Non Destructive Readout

**Modulation of the time constant**
- Logarithmic response
- Lin-log

**Modulation of the integration capacitance**
- CCD with two wells
- Overflow MOSFET capacitors
- Adding multiple shorter integration periods
- Smart reset pixels

“dynamic range > 80 dB cannot be reached with a linear response sensor” – so what?
Non-linear version of $V_{\text{signal}}(Q)$

$$V_{\text{signal}} = \frac{t_{\text{int}} \cdot I_{\text{photo}}}{C_{\text{eff}}}$$ (linear)

$$V_{\text{signal}} = \sum t_{\text{int}} \cdot \frac{I_{\text{photo}}}{C_{\text{eff}}} = \frac{Q_{\text{photo}}}{t_{\text{int max}}} \cdot \sum \frac{t_{\text{int}}(Q)}{C_{\text{eff}}(Q)}$$

$$V_{\text{signal}} = \frac{1}{t_{\text{int max}}} \cdot \int_{0}^{Q_{\text{photo}}} \frac{t_{\text{int}}(Q)}{C_{\text{eff}}(Q)} dQ$$

- We assume that the total photo charge is cut in pieces
- Each piece may have a different integration time or integration capacitor.
- Think on a charge accumulation axis, not on a time axis

$$\text{slope} = \frac{t_{\text{int}}(Q)}{C_{\text{eff}}(Q)}$$
Non-linear version of $V_{\text{signal}}(Q)$ (2)

$$V_{\text{signal}} = \frac{t_{\text{int}} \cdot I_{\text{photo}}}{C_{\text{eff}}}$$  (linear)

$$V_{\text{signal}} = \sum t_{\text{int}} \cdot \frac{I_{\text{photo}}}{C_{\text{eff}}} = \frac{Q_{\text{photo}}}{t_{\text{int max}}} \cdot \sum \frac{t_{\text{int}}(Q)}{C_{\text{eff}}(Q)}$$

slope = $t_{\text{int}}(Q)/C_{\text{eff}}(Q)$
Is non-linearity mandatory?

Non-linearity (non-linear response) is *not* essential. (e.g. *Non-Destructive Readout* is a linear method)

It may be considered as a feature of the acquisition process, helping to catch the wide range of intensities in the image. It *may* do so by taking noise levels (NEC!) into account.

In a post-processing step the non-linearity may be linearized.
Formula for NEC in charge domain

\[
NEC = \frac{P}{NEP} = \frac{P \cdot \frac{\partial S}{\partial P}}{N} \]

\[P \sim Q_{\text{photo}} \text{ total photo charge during frame time}\]
\[Q_{\text{signal}} \text{ charge integrated during } t_{\text{int}}\]
\[N \sim Q_{\text{noise}} \text{ uncertainty on } Q_{\text{signal}}\]
\[Q_{\text{signal}} / Q_{\text{photo}} = t_{\text{int}} / t_{\text{int, max}}\]

\[\Rightarrow NEC = \frac{Q_{\text{photo}} \cdot \frac{\partial Q_{\text{signal}}}{\partial Q_{\text{photo}}}}{Q_{\text{noise}}}\]

\[\Rightarrow NEC = \frac{Q_{\text{photo}} \cdot \frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \cdot \frac{Q_{\text{photo}}}{t_{\text{int, max}}}}{Q_{\text{noise}}}\]

\[NEC \text{ does not depend on } C_{\text{eff}} \text{ in charge domain}\]
Formula for NEC in voltage domain

\[
NEC = \frac{P}{NEP} = \frac{P \cdot \frac{\partial S}{\partial P}}{N}
\]

\[
V_{signal} = S
\]

\[
V_{noise} = N
\]

\[
SR = \text{Spectral Response [A/W]}
\]

\[
V_{signal} = \int t_{int} \cdot \frac{I_{photo}}{C_{eff}} = \int t_{int} \cdot \frac{P \cdot SR}{C_{eff}}
\]

\[
\frac{\partial S}{\partial P} = SR \left( \frac{t_{int}}{C_{eff}} + \frac{\partial t_{int} \cdot P}{C_{eff}} - \frac{t_{int} \cdot P \cdot \frac{\partial C_{eff}}{\partial P}}{C_{eff}^2} \right)
\]

When one lets integration time depend on P or Q\text{photo}

\[
NEC = \frac{P}{N} \cdot \frac{\partial S}{\partial P} = \frac{t_{int} \cdot P}{N} + \frac{t_{int} \cdot P^2 \cdot \frac{\partial C_{eff}}{\partial P}}{C_{eff}^2}
\]

When one lets C\text{eff} depend on P or Q\text{photo}
Keep NEC constant... in the presence of a fixed noise charge

\[ NEC = \frac{\frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}}}{\frac{\partial Q_{\text{photo}}}{t_{\text{int,max}}} Q_{\text{noise}}} \]

\[ Q_{\text{noise}} = \text{constant} \]

\[ \frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \sim \frac{1}{Q_{\text{photo}}^2} \]

\[ t_{\text{int}}(Q_{\text{photo}}) \sim \frac{1}{Q_{\text{photo}}} \]

\[ NEC = \frac{Q_{\text{signal}}}{Q_{\text{noise}}} = \frac{Q_{\text{photo}}}{Q_{\text{noise}}} \]

No solution

i.e. there is no \( C_{\text{eff}}(P) \) law that results in a constant NEC when noise a fixed charge. (This does not prohibit the use of \( C_{\text{eff}} \) modulation in general)
Keep NEC constant…
in the presence of kTC noise

\[ t_{\text{int}} \text{ varies} \]

\[ \frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \cdot \frac{Q_{\text{photo}}}{2} \]

\[ NEC = \frac{\partial Q_{\text{photo}}}{\partial t_{\text{int max}}} \cdot \frac{Q_{\text{noise}}}{Q_{\text{noise}}} \]

\[ Q_{\text{noise}} = \text{constant (!)} \]

\[ \frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \sim \frac{1}{Q_{\text{photo}}^2} \]

\[ t_{\text{int}} (Q_{\text{photo}}) \sim \frac{1}{Q_{\text{photo}}} \]

\[ C_{\text{eff}} \text{ varies} \]

\[ NEC = \frac{Q_{\text{photo}}}{Q_{\text{noise}}} \]

\[ Q_{\text{noise}} = \sqrt{kTC_{\text{eff}}} \sim \sqrt{C_{\text{eff}}} \]

\[ C_{\text{eff}} \sim Q_{\text{photo}}^2 \]
Keep NEC constant... in the presence of photon shot noise

\[ t_{\text{int}} \text{ varies} \]

\[
NEC = \frac{\frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \cdot Q_{\text{photo}}^2}{\frac{\partial Q_{\text{photo}}}{t_{\text{int max}}} \cdot Q_{\text{noise}}} \]

\[ Q_{\text{noise}} \sim \sqrt{Q_{\text{signal}}} \sim \sqrt{Q_{\text{photo}}} \cdot t_{\text{int}} \]

\[
\frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \sim \frac{\sqrt{Q_{\text{photo}}} \cdot \sqrt{t_{\text{int}}}}{Q_{\text{photo}}^2} \]

\[ t_{\text{int}} (Q_{\text{photo}}) \sim \frac{1}{Q_{\text{photo}}} \]

\[ C_{\text{eff}} \text{ varies} \]

\[ NEC = \frac{Q_{\text{photo}}}{Q_{\text{noise}}} \]

\[ Q_{\text{noise}} \sim \sqrt{Q_{\text{photo}}} \]

No solution

i.e. there is no \( C_{\text{eff}}(P) \) law that results in a constant NEC when noise is exclusively PSN. This does not prohibit the use of \( C_{\text{eff}} \) modulation in general.
Keep NEC constant... when dominated by PRNU

\[
NEC = \frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \cdot \frac{Q_{\text{photo}}^2}{Q_{\text{noise}}} \cdot \frac{t_{\text{int max}}}{Q_{\text{noise}}}
\]

\[
Q_{\text{noise}} \sim Q_{\text{signal}} \sim Q_{\text{photo}} \cdot t_{\text{int}}
\]

\[
\frac{\partial t_{\text{int}}(Q)}{\partial Q_{\text{photo}}} \sim \frac{t_{\text{int}}(Q)}{Q_{\text{photo}}}
\]

\[
C_{\text{eff}} \text{ varies}
\]

\[
NEC = \frac{Q_{\text{photo}}}{Q_{\text{noise}}}
\]

\[
\text{PRNU} = \frac{Q_{\text{noise}}}{Q_{\text{photo}}} = \text{constant}
\]

Always fulfilled

i.e., for any relation \(C_{\text{eff}}(P)\), NEC is constant for systems dominated by PRNU
Keep NEC constant in the presence of a fixed voltage noise

\[ t_{\text{int}} \text{ varies} \]

\[
NEC = \frac{SR \cdot \left( \frac{\partial t_{\text{int}} \cdot P^2}{\partial P} \right)}{N}
\]

\[ V_{\text{noise}} = \text{constant} \sim \frac{\partial t_{\text{int}}}{\partial P} \cdot P^2 \]

\[
\frac{\partial t_{\text{int}}}{\partial P} \sim \frac{1}{P^2}
\]

\[ t_{\text{int}} (P) \sim \frac{1}{P} \]

\[ C_{\text{eff}} \text{ varies} \]

\[
NEC = \frac{SR \cdot \left( t_{\text{int}} \cdot P^2 \cdot \frac{\partial C_{\text{eff}}}{\partial P} \right)}{N}
\]

\[ V_{\text{noise}} = \text{constant} \sim \frac{P^2 \cdot \partial C_{\text{eff}}}{C_{\text{eff}}^2} \]

\[
\frac{\partial C_{\text{eff}}}{\partial P} \sim \frac{C_{\text{eff}}^2}{P^2}
\]

\[ C_{\text{eff}} \sim P \]
Keep NEC constant... when dominated by dark current shot noise

\[ t_{\text{int}} \text{ varies} \]

\[ NEC = \frac{\frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \cdot Q_{\text{photo}}^2}{\frac{\partial Q_{\text{photo}}}{Q_{\text{noise}}} \cdot t_{\text{int max}}} = \frac{Q_{\text{noise}}}{Q_{\text{photo}}} \cdot \sqrt{t_{\text{int}}} \]

\[ Q_{\text{noise}} \sim \sqrt{t_{\text{int}}} \]

\[ \frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \sim \sqrt{t_{\text{int}}} \]

\[ \frac{\partial Q_{\text{photo}}}{Q_{\text{photo}}} \sim \sqrt{Q_{\text{photo}}} \]

\[ t_{\text{int}} (Q_{\text{photo}}) \sim \frac{1}{Q_{\text{photo}}^2} \]

\[ C_{\text{eff}} \text{ varies} \]

\[ NEC = \frac{Q_{\text{photo}}}{Q_{\text{noise}}} \]

\[ Q_{\text{noise}} \sim \text{constant} \]

No solution
Keep NEC constant…
when dominated by DSNU

\[
NEC = \frac{\frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \cdot Q_{\text{photo}}^2}{\frac{\partial Q_{\text{photo}}}{\partial t_{\text{int}}} \cdot t_{\text{int max}}} \frac{Q_{\text{noise}}}{Q_{\text{noise}}}
\]

\[Q_{\text{noise}} \sim t_{\text{int}}\]

\[
\frac{\partial t_{\text{int}}}{\partial Q_{\text{photo}}} \sim \frac{t_{\text{int}}}{Q_{\text{photo}}^2}
\]

\[C_{\text{eff}} \text{ varies}\]

\[NEC = \frac{Q_{\text{photo}}}{Q_{\text{noise}}}\]

\[Q_{\text{noise}} \sim \text{constant}\]

No solution
### Summary of “keep NEC constant”

#### Nature of noise

Noise sources that persist after calibration in high end imagers

<table>
<thead>
<tr>
<th>Nature of noise</th>
<th>kTC noise</th>
<th>PSN</th>
<th>DCSN</th>
<th>PRNU</th>
<th>DSNU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant charge&lt;sub&gt;RMS&lt;/sub&gt;</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>always</td>
<td>No solution</td>
</tr>
<tr>
<td>Constant voltage&lt;sub&gt;RMS&lt;/sub&gt;</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>always</td>
<td>No solution</td>
</tr>
<tr>
<td>~sqrt(power)</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>always</td>
<td>No solution</td>
</tr>
<tr>
<td>~sqrt(t&lt;sub&gt;int&lt;/sub&gt;)</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>t&lt;sub&gt;int&lt;/sub&gt; ~ 1/Q</td>
<td>always</td>
<td>No solution</td>
</tr>
<tr>
<td>~power</td>
<td>C&lt;sub&gt;eff&lt;/sub&gt; ~ Q&lt;sup&gt;2&lt;/sup&gt;</td>
<td>C&lt;sub&gt;eff&lt;/sub&gt; ~ Q</td>
<td>No solution</td>
<td>always</td>
<td>No solution</td>
</tr>
<tr>
<td>~t&lt;sub&gt;int&lt;/sub&gt;</td>
<td>C&lt;sub&gt;eff&lt;/sub&gt; ~ Q</td>
<td>C&lt;sub&gt;eff&lt;/sub&gt; ~ Q</td>
<td>No solution</td>
<td>always</td>
<td>No solution</td>
</tr>
</tbody>
</table>

### Mathematical Formulas

\[
NEC = \frac{Q_{\text{photo}}}{Q_{\text{noise}}}
\]

\[
NEC_{\text{photo}} = \frac{\frac{\partial Q_{\text{signal}}}{\partial P} \cdot P^2}{\frac{\partial Q_{\text{photo}}}{\partial P} \cdot C_{\text{eff}}}
\]

\[
NEC_{\text{eff}} = \frac{Q_{\text{photo}}}{Q_{\text{noise}}}
\]

\[
NEC = \frac{t_{\text{int}} \cdot P^2 \cdot \frac{\partial C_{\text{eff}}}{\partial P} \cdot C_{\text{eff}}}{N}
\]

\[
SR = \frac{\frac{\partial t_{\text{int}}}{\partial P} \cdot P^2}{\frac{\partial C_{\text{eff}}}{\partial P} \cdot C_{\text{eff}}}
\]

### Relevant Noise Terms

- Noise sources that persist after calibration in high end imagers
- EMI, read noise, ADC...
- kTC noise
- PSN
- DCSN
- PRNU
- DSNU

### Observations

- Constant charge<sub>RMS</sub>
- Constant voltage<sub>RMS</sub>
- ~sqrt(power)
- ~sqrt(t<sub>int</sub>)
- ~power
- ~t<sub>int</sub>

### Additional Notes

- Noise varies with respect to integration time and effective capacitance.
- The nature of noise depends on the specific noise source and its characteristics.
One can define a $t_{\text{int}}(Q)$ or $C_{\text{eff}}(Q)$ law to obtain a constant and optimal NEC for most common noise signatures.

The relations $t_{\text{int}} \sim 1/Q$ and $C_{\text{eff}} \sim Q$ found are essentially “logarithmic responses”.

\[
V_{\text{signal}} = \frac{1}{t_{\text{int \ max}}} \cdot \int_0^{Q_{\text{photo}}} \frac{t_{\text{int}}(Q)}{C_{\text{eff}}(Q)} dQ \sim \int_0^{Q_{\text{photo}}} \frac{1}{Q} dQ
\]

\[
V_{\text{signal}} \sim \log n (Q_{\text{photo}}) + \text{Cte}
\]

For $t_{\text{int}} \sim 1/P^2$ and $C_{\text{eff}} \sim P^2$:

\[
V_{\text{signal}} \sim \int_0^{Q_{\text{photo}}} \frac{1}{Q^2} dQ
\]

\[
V_{\text{signal}} \sim \frac{1}{Q_{\text{photo}}} + \text{Cte}
\]
Linear response
Logarithmic response for NEC ≈ 10
Approximation multiple slopes for NEC ≥ 10

Illumination, impinging photons

V_{out}
Result NEC ≥ 10

Noise sources considered:
• PSN
• Read noise 1 mV_{RMS}

Essential pixels specs
• QE = 40%
• C_{eff} = 1 fF
• t_{int} = not specified

Full well limit ≈ 1 fF \times 1 V = 6000 \text{e}^-
15000 photons
Linear response

Noise sources considered:
- PSN
- Read noise $1\text{mV}_{\text{RMS}}$

Essential pixels specs:
- $\text{QE} = 40\%$
- $C_{\text{eff}} = 1\text{fF}$
- $t_{\text{int}} = \text{not specified}$

Full well limit $\approx 1\text{fF} \times 1\text{V} \approx 6000 \text{e}^-$
15000 photons
Linear response

One single integration time

- NEP [photons]
- NEC
- integrated photons
- impinging photons

#photons impinging on a pixel

Full well limit!
Noise sources considered:
• PSN
• Read noise $1\text{mV}_{\text{RMS}}$

QE = 40%
$C_{\text{eff}} = 1\text{fF}$
$t_{\text{int}} = \text{not specified}$

(Logarithmic) response realized by continuously reducing the integration time during the charge acquisition, to have $NEC \approx 10$
Logarithmic response NEC\approx10

continuously varying integration time \( \text{t}_{\text{int}}(Q) \)

- NEP [photons]
- NEC
- integrated photons
- impinging photons

\( \text{t}_{\text{int}} > \text{t}_{\text{int max}} \) ? NEP is limited to linear case
Multiple slopes methods NEC>10

Noise sources considered:
- PSN
- Read noise 1mV_{RMS}

QE = 40%
C_{eff}=1fF
\text{t}_{int}=\text{not specified}

Multiple slopes
- 1^{st} slope starts at 10000 photons (4000 electrons)
- Consecutive slopes have factor 10 lower integration time
- New slope starts only when NEC\geq10 is assured
Multiple slopes methods NEC>10

Multiple slopes using multiple integration times

- NEP [photons]
- NEC
- integrated photons
- impinging photons

#photons impinging on a pixel
Conclusions
Was there something you might want to remember?

- High (wide) dynamic range is a property of the scene. The sensor has to accommodate.
- How to accommodate: yielding a sufficient NEC in all parts of the image/scene.
- The logarithmic response follows naturally when assuming a typical noise signature, and aiming for a constant, minimal NEC.
- Piece-wise approximation (multiple slopes) of the ideal logarithmic response is not as good.
- How to implement? 😊 the next presentations.
Thank you
**Abbreviations and symbols**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{eff}}$</td>
<td>charge to voltage transconductance</td>
</tr>
<tr>
<td>D</td>
<td>lens aperture</td>
</tr>
<tr>
<td>DCSN</td>
<td>dark current shot noise</td>
</tr>
<tr>
<td>DSNU</td>
<td>dark signal non uniformity</td>
</tr>
<tr>
<td>EMI</td>
<td>electro-magnetic interference</td>
</tr>
<tr>
<td>f</td>
<td>focal length</td>
</tr>
<tr>
<td>F</td>
<td>f-number $F=f/D$</td>
</tr>
<tr>
<td>FPN</td>
<td>fixed pattern noise</td>
</tr>
<tr>
<td>N</td>
<td>Noise, uncertainty on $S$</td>
</tr>
<tr>
<td>NEC</td>
<td>noise equivalent contrast [%]</td>
</tr>
<tr>
<td>NEP</td>
<td>noise equivalent power [W...]</td>
</tr>
<tr>
<td>P</td>
<td>optical power [W, photons/s, …]</td>
</tr>
<tr>
<td>PR</td>
<td>photo response [V/W, V.m²/W.s]</td>
</tr>
<tr>
<td>PRNU</td>
<td>PR non-uniformity</td>
</tr>
<tr>
<td>$Q_{\text{noise}}$</td>
<td>uncertainty on $Q_{\text{signal}}$ [F, e⁻]</td>
</tr>
<tr>
<td>$Q_{\text{photo}}$</td>
<td>(maximum) photo charge [F, e⁻]</td>
</tr>
<tr>
<td>$Q_{\text{signal}}$</td>
<td>actual photo charge, due to electronic shutter</td>
</tr>
<tr>
<td>QE</td>
<td>quantum efficiency [electrons/photon]</td>
</tr>
<tr>
<td>RMS</td>
<td>root-mean-square, subscript denoting that the value is a distribution</td>
</tr>
<tr>
<td>S</td>
<td>Signal, normally equal to $V_{\text{signal}}$</td>
</tr>
<tr>
<td>SR</td>
<td>spectral response [A/W]</td>
</tr>
<tr>
<td>$t_{\text{int}}$</td>
<td>integration time, electronic shutter time</td>
</tr>
<tr>
<td>$t_{\text{intmax}}$</td>
<td>maximum available integration time</td>
</tr>
<tr>
<td>$V_{\text{noise}}$</td>
<td>uncertainty on $V_{\text{signal}}, = N$</td>
</tr>
</tbody>
</table>